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SCALAR FUZZY LOGIC
A NEW MATHEMATIC MODEL FOR APPROXIMATE REASONING

M. F. Mlynski[1], W. Ameling[2]
[1]CWA GmbH, Aachen, Germany
[2]Department of Electical Engineering and Computer Science, University of Aachen, Aachen, Germany

Abstract-**Atrial and ventricular tachycardia are typically treated with Implantable Cardioverter Defibrillators (ICDs) [8]. Current dual-chamber ICDs have over 200 parameters, which have to be set all properly to ensure an accurate ICD programming. Since this is a time-consuming procedure, an expert-system to calculate a complete set of ICD parameters based on the given clinical patient-data was developed, where the expert knowledge was acquired and implemented into a knowledge base in cooperation with cardiologist physicians [1].**
***Keywords* - Expert System, Fuzzy Logic**

I. INTRODUCTION

Since human expert knowledge is not always precise, fuzzy logic [9] had to be used to implement the human expert knowledge. Using the conventional fuzzy logic showed up with several problems. Using fuzzy-sets and relation-matrices requires high calculation power. Further more the use of linguistic variables and fuzzy-sets are difficult to understand for a physician. From this the physician has almost no possibility to verify the knowledge base and prove its correctness. Last but not least the maintenance cost are quite high: When redefining, adding or deleting fuzzy sets of a linguistic variable, all relation-matrices using this linguistic variable within the knowledge base have to be verified and perhaps corrected. All these problems are most likely for a knowledge base with several hundreds up to few thousand rules.

This article presents a new fuzzy technique, called the Scalar Fuzzy Logic (SFL), which does without linguistic variables, fuzzy-sets and relation-matrices. It uses scalar variables and scalar fuzzy operators instead. The SFL is a systematic extension of Boolean logic which avoids all problems mentioned above. First investigations with an expert system for the Tachos-ICD from BIOTRONIK (Germany) have proved that human expert knowledge can be easily implemented and maintained using the SFL and calculation time is very low.

II. EXPERT SYSTEMS AND FUZZY LOGIC

One of the essential points of Expert Systems (ES) is the knowledge base, since all the acquired expert expertise and experience is implemented into the knowledge base. Typical types of knowledge representation are fact-based, rule-based and frame-based (including semantic networks) [3], [6], [7].

Since fact-based ES showed up to be to limited to implement the medical knowledge we decided to use a rule-based knowledge representation. In order to implement unprecise knowledge, first the fuzzy logic [9] was used for the ES.

Using the fuzzy logic [2], [4], input data are represented by so called linguistic variables, with each linguistic variable owning several so called fuzzy sets. A fuzzy set is an (unprecise) description of a statement, e.g. "age of patient is young". All (real) input data are fuzzified into the fuzzy sets of the corresponding linguistic variable, which is called fuzzification. As an example, fuzzifying the age of a patient, a linguistic variable *age_of_patient* has to be defined first. *age_of_patient* may have 4 fuzzy-sets: *very_young*, *young*, *old* and *very_old*. Thus, a real age in years will be fuzzified into a fuzzy-vector, depending on the defined member-functions for each fuzzy-set:

$$a \xrightarrow{F} \vec{a} = \begin{pmatrix} a_1 & a_2 & a_3 & a_4 \end{pmatrix}^T. \quad (1)$$

Rules are implemented using the fuzzy-inference. While the fuzzified input-data are represented by the fuzzy-vectors [5], the rule conditions are implemented into so called inference matrices. The result is again a fuzzy-vector with the (unprecise) statement of the rule conclusion:

$$\vec{r}^T = \vec{a}^T \otimes A \quad (2)$$

As an example, the rule-condition "*patient is young* AND *patient is active*" is implemented by:

$$\left(\vec{a}^T \otimes A\right) \otimes_{AND} \left(\vec{b}^T \otimes B\right) \quad (3)$$

where $\vec{b}$ is the fuzzy-vector of the fuzzified activity of the patient, a classification rated from 0 (very inactive) up to 100 (very active). A complete rule, e.g. to "calculate" the UTR (upper tracking rate) of an ICD, may be:

Example 1

Rule to determine UTR:

IF *patients age is medium or young*
AND *activity is high*
AND *NYHA-class is low*
THEN *UTR should be high*

We define four linguistic variables *age_of_patient*, *activity_of_patient*, *NYHA_class_of_patient* and *UTR*. We define 4 fuzzy-sets for *age_of_patient*, 5 fuzzy-sets for *activity_of_patient*, 4 fuzzy-sets for *NYHA_class_of_patient* and 6 fuzzy-sets for the result *UTR*. The rule is implemented using the fuzzy-logic by:

$$\vec{u} = \left(\vec{a}^T \otimes A\right) \otimes_{AND} \left(\vec{b}^T \otimes B\right) \otimes_{AND} \left(\vec{c}^T \otimes C\right) =$$

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$$\left(\begin{pmatrix} a_1 & a_2 & a_3 & a_4 \end{pmatrix} \otimes \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & \ddots & \ddots & \ddots & \ddots & \vdots \\ a_{31} & \ddots & \ddots & \ddots & \ddots & \vdots \\ a_{41} & \cdots & \cdots & \cdots & \cdots & a_{46} \end{pmatrix} \right) \otimes_{AND}$$
$$\left(\begin{pmatrix} b_1 & b_2 & b_3 \end{pmatrix} \otimes \begin{pmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} & b_{16} \\ b_{21} & \ddots & \ddots & \ddots & \ddots & \vdots \\ b_{31} & \ddots & \ddots & \ddots & \ddots & \vdots \\ b_{41} & \ddots & \ddots & \ddots & \ddots & \vdots \\ b_{51} & \cdots & \cdots & \cdots & \cdots & b_{56} \end{pmatrix} \right) \otimes_{AND}$$
$$\left(\begin{pmatrix} c_1 & c_2 & c_3 & c_4 \end{pmatrix} \otimes \begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & \ddots & \ddots & \ddots & \ddots & \vdots \\ c_{31} & \ddots & \ddots & \ddots & \ddots & \vdots \\ c_{41} & \cdots & \cdots & \cdots & \cdots & c_{46} \end{pmatrix} \right) . \quad (4)$$

The result of the fuzzy-inference, the fuzzy-vector $\vec{u}$ for the UTR, have to be transformed back into a real value (e.g. 120 bpm = 120 beats per minute).This transform of a fuzzy-vector into a corresponding real value is called defuzzification. The most common methods are the center of gravity and the approximated center of gravity method [4].

III. Disadvantages Of The Fuzzy Logic

Implementing the knowledge base for the ICD programming showed up with several problems when using the "conventional" fuzzy logic:

1. The "transform" of the textual rule condition into the fuzzy matrices is sometimes difficult, especially when the fuzzy-matrices are not quadratic.
2. The more fuzzy-sets for each linguistic variable are used, the better results are achieved. But simultaneously the calculation time increases.
3. Using less fuzzy-sets for each linguistic variable results in acceptable calculation time, but the inference results are often not accurate enough.
4. Every time one or several fuzzy-set of a linguistic variable are changed, every single rule of the whole knowledge base using this linguistic variable has to be adjusted, i.e. the relations-matrices have to be reviewed and possibly changed.
5. For the human experts, who are typically not familiar with vector and matrix analysis, especially not with the fuzzy-inference, it is almost impossible to verify and prove the implemented knowledge.

Implementing a knowledge base with a couple or a few dozen rules, most of the problems mentioned above can be solved. For the ES for programming an ICD the knowledge base rapidly grew up to several 100 rules and most of the problems mentioned above showed up to be insoluble.

IV. The Scalar Fuzzy Logic

From the problems showing up when using the "conventional" fuzzy logic we have developed a new kind of fuzzy logic which does without linguistic variables, fuzzy-sets and relation-matrices. It uses scalar variables and scalar fuzzy operators instead and is called "scalar fuzzy logic" (SFL) therefore. The SFL is a systematic extension of the Boolean logic and includes unprecise operations.

First we introduce fuzzy comparison operators to implement unprecise conditions. With m>0 we define:

Fuzzy Equal: $$x \overset{FE}{=} y \;:\; FE(x,y) = \frac{1}{1+(x-y)^2 \cdot m} \quad (5)$$

Fuzzy Not Equal: $$x \overset{FNE}{\neq} y \;:\; FNE(x,y) = 1 - \frac{1}{1+(x-y)^2 \cdot m} \quad (6)$$

Fuzzy Less: $$x \overset{FL}{<} y \;:\; FL(x,y) = \frac{1}{2} \cdot \left(\frac{(y-x)}{|y-x|+m} + 1 \right) \quad (7)$$

Fuzzy Greater: $$x \overset{FG}{>} y \;:\; FG(x,y) = \frac{1}{2} \cdot \left(\frac{(x-y)}{|x-y|+m} + 1 \right) \quad (8)$$

It can be proved that:

1. The result r is always in-between 0 and 1, where 0 indicates that the comparison is totally not true and 1 indicates that the comparison is totally true:
2. FE(x,y) → 0 for $x >> y$
   FE(x,y) → 0 for $x << y$
   FE(x,y) = 1 for $x = y$
   FNE(x,y) → 1 for $x >> y$
   FNE(x,y) → 1 for $x << y$
   FNE(x,y) = 0 for $x = y$
   FL(x,y) → 0 for $x >> y$
   FL(x,y) → 1 for $x << y$
   FG(x,y) → 0 for $x << y$
   FG(x,y) → 1 for $x >> y$
3. All fuzzy comparison operators represent continuously differentiable functions in $\Re^2$.

For combining several rule conditions, the following fuzzy combination operators are introduced. With $M_{AND} \geq n\cdot(n-1)$ and $M_{OR} \geq n\cdot(n-1)$ we define:

$$AND_{fuzzy}(p_1, p_2, \ldots p_n) = MIN(p_1, p_2, \ldots p_n) + \frac{\sum_{n1=1}^{n-1} \sum_{n2=n1+1}^{n} |p_{n1} - p_{n2}|}{M_{AND}} \quad (9)$$

$$OR_{fuzzy}(p_1, p_2, \ldots p_n) = MAX(p_1, p_2, \ldots p_n) - \frac{\sum_{n1=1}^{n-1} \sum_{n2=n1+1}^{n} |p_{n1} - p_{n2}|}{M_{OR}} \quad (10)$$

It can be proved that:

1. With every $p_i \in [0,1]$, i=1..n, the result r is always in-between 0 and 1, where 0 indicates that the combination-result is totally not true and 1 indicates that the combination-result is totally true.

Using the SFL the rule from Example 1 to determine the UTR is implemented by:

$$UTR = 120 + 30 * \left(\left(a \overset{FL}{<} 60 \right) AND_{fuzzy} \left(b \overset{FG}{>} 65 \right) AND_{fuzzy} \left(c \overset{FL}{<} 2 \right) \right) \quad (11)$$

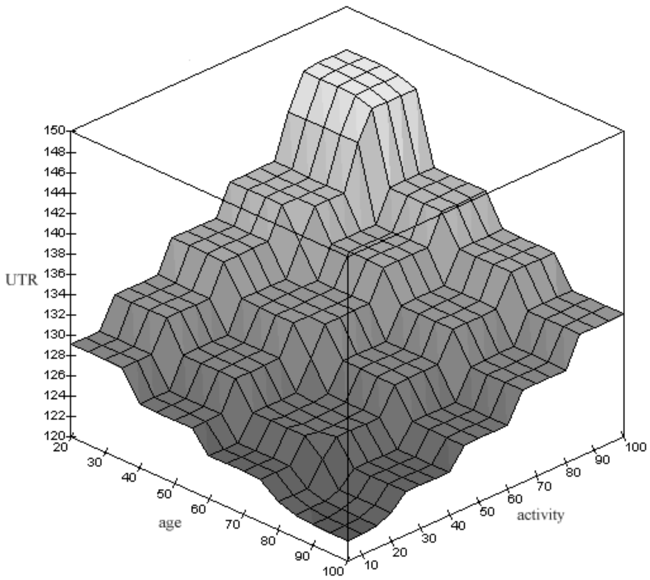

Fig. 1. Result of Example 1 with NYHA = 1,
using "conventional" fuzzy logic according to equation (4)

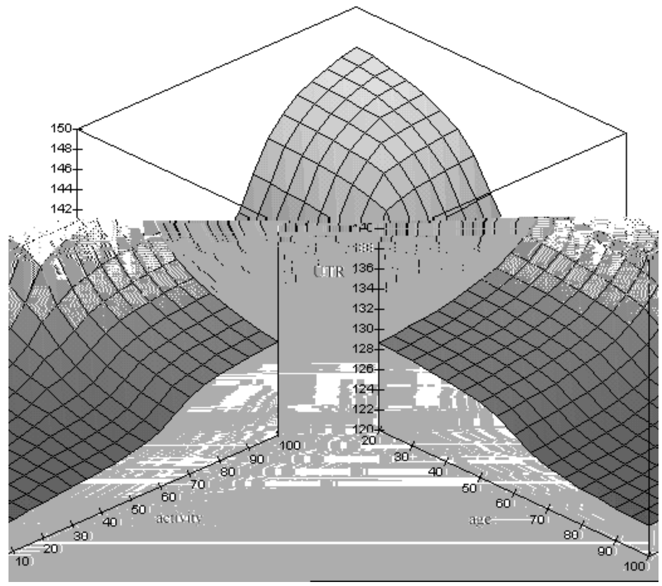

Fig. 2. Result of Example 1 with NYHA = 1,
using the scalar fuzzy logic according to equation (11)

which is more easy to implement, less computing power is needed for calculation, any changes on this specific rule has no side affects on any other rules within the knowledge base and the implementation can be verified and proved by the human experts much more easily than the implementation due to equation (4).

Figure 1 shows the result of Example 1 with NYHA = 1, for all ages from 20 years up to 100 years and all activity levels from 1 up to 100, using the implementation of equation (4), where the fuzzy-sets where defined by triangle membership functions and the approximate center of gravity method was used for defuzzification. Figure 2 shows the result for the very same rule but implemented with the SFL according to equation (11).

V. Discussion

We suggest a new type of fuzzy logic to implement unprecise reasoning, which shows great advantages on building expert systems with huge knowledge bases. Further investigations will show the practical usability of the SFL.

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